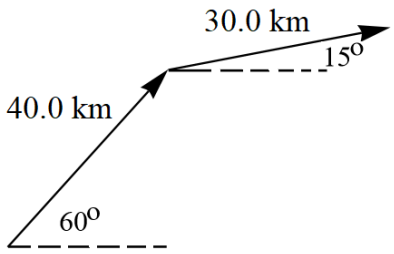
Problem 50

A small plane flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east. Use the analytical method to find the total distance the plane covers from the starting point, and the geographic direction of its displacement vector. What is its displacement vector?

Solution

Draw a schematic of the plane's path.



The plane's displacement vector is the sum of the two vectors shown above.

$$\begin{aligned}\mathbf{d} &= (40.0\cos 60°\hat{\mathbf{i}} + 40.0\sin 60°\hat{\mathbf{j}})\text{ km} + (30.0\cos 15°\hat{\mathbf{i}} + 30.0\sin 15°\hat{\mathbf{j}})\text{ km} \\ &= (40.0\cos 60° + 30.0\cos 15°)\hat{\mathbf{i}}\text{ km} + (40.0\sin 60° + 30.0\sin 15°)\hat{\mathbf{j}}\text{ km} \\ &\approx (49.0\text{ km})\hat{\mathbf{i}} + (42.4\text{ km})\hat{\mathbf{j}}\end{aligned}$$

The geographic direction of this vector is

$$\theta = \tan^{-1}\left(\frac{40.0\sin 60° + 30.0\sin 15°}{40.0\cos 60° + 30.0\cos 15°}\right) \approx 40.9°\text{ north of east.}$$

The total distance the plane covers from the starting point is the magnitude of the plane's displacement vector.

$$\begin{aligned}|\mathbf{d}| &= \sqrt{(40.0\cos 60° + 30.0\cos 15°)^2 + (40.0\sin 60° + 30.0\sin 15°)^2}\text{ km} \\ &\approx \sqrt{(49.0)^2 + (42.4)^2}\text{ km} \\ &\approx 64.8\text{ km}\end{aligned}$$